Int. J. Heat Mass Transfer. Vol. 33, No. 9, pp. 1829–1837, 1990
Printed in Great Britain

0017-9310/90 $3.00 + 0.00

Response of a solid–gas growth interface to a homogeneous time dependent acceleration field

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(*Received* 8 *February* 1989 *and in final form* 9 *October* 1989)

Abstract—The matched asymptotic expansions technique is used to analyse the core and boundary layer solution of the compressible unsteady viscous one-dimensional Navier–Stokes equations in order to study the transport at a growth interface due to a homogeneous, sinusoidal in time and slowly varying weak gravity field. It is shown that: (a) the mass transfer at the interface is governed by the time dependence of the specific mass at the isothermal active interface; (b) diffusion occurs in a thin boundary layer in front of the interface under the form of a damped travelling concentration wave; (c) unless very small growth rates are considered as in some epitaxial growth processes, g-jitters as encountered onboard spacecrafts have negligible effects on mass transfer at a solid–gas growth interface.

1. INTRODUCTION

DUE TO the strong decrease of buoyant convection, second-order driving forces may drive significant fluid motion in nearly zero-gravity conditions. In gases, fluid motion can be generated by thermomechanical couplings caused by the compressibility of the medium. The hydrodynamic modelling of these problems is based on the low speed compressible Navier–Stokes equations the solution of which have needed new numerical methods to be built [1, 2]. These non-divergence free velocity fields caused by the expansion of compressible layers play a very important role in transient heat transfer for which asymptotic methods are very useful to extract the physics of the complex mechanisms which drive the processes. For example, Kassoy has used such techniques to explore the response of a perfect gas contained in a one-dimensional slot to heat addition at one boundary [3–6].

Moreover, it has also been recently shown that significant fluid motion could be generated by local unsteady heat addition [7] showing that transient mass and heat transfer in gases often deals with thermoacoustics and compressibility, even at very low speed.

All these studies have been devoted to thermally driven thermomechanical disturbances, that is to say situations for which the transfer of energy goes from the thermal energy to the kinetic energy through compressibility. The problem under study in the present paper is that in which the energy is brought to the gas through mechanical disturbances caused by time dependent homogeneous weak gravity perturbations. More precisely, the purpose of this study is to explore through a one-dimensional model and analytical methods the effects of such mechanical disturbances on a growth interface in order to model the effects of gravity perturbations on crystal growth experiments performed onboard spacecrafts in low earth orbit.

To this end, a one-dimensional slot is considered filled with a binary gas mixture, one component of which may undergo a phase change at the interface located at $x = 0$.

A solution of the one-dimensional unsteady Navier–Stokes equations coupled with the diffusion equation is looked for on the time scale of the perturbation by means of the matched asymptotic expansion technique. The solutions for density, temperature, velocity, pressure and weight fraction are given in the case of a sinusoidal homogeneous gravity perturbation, the characteristic time of which is in between the short acoustic scale and the long diffusive one.

The first part is devoted to the pure gas approach without the interface while the binary gas mixture with the growing interface is treated in the second part.

2. MONOCOMPONENT GAS AND NON-CATALYTIC WALLS

2.1. *The model and governing equations*

We consider a one-dimensional slot of width L filled with a perfect gas B and supposed to be in quasi zero-gravity conditions (Fig. 1). Both ends ($x' = 0$ and L) are at an imposed temperature T_0. For $t < 0$ the system is at rest and in thermal equilibrium. For $t \geqslant 0$ a homogeneous time dependent acceleration field is applied in the x-direction. The amplitude is $10^{-2}g_0$ (where g_0 is the ground value of the gravity acceleration) and the frequency is 50 Hz. These conditions are roughly those encountered onboard spacecrafts in low earth orbit. The motion of the gas is described by the one-dimensional Navier–Stokes equations. If the length is reported to the length of the domain, the time to the acoustic time $t'_a = L/C'_0$ where C'_0 is the speed of sound in the initial state, and the other dependent variables reported to their initial value, the governing equations can be written as

NOMENCLATURE

A	amplitude of the momentum source term
C'_0	sound velocity in the reference state
D	diffusion coefficient
L	width of the one-dimensional slot
Le	Lewis number
M_x	molar mass of specie x
P	pressure
Pr	Prandtl number, κ/ν
S	momentum source term
t'_a	acoustic characteristic time
t'_d	diffusive characteristic time
t	time normalized with respect to the acoustic time
$\bar{t}$	time normalized with respect to the characteristic time of the perturbation
T	temperature
T_0	reference temperature
u_0	initial velocity
W	weight fraction of specie A
W_0	initial value of the weight fraction of A
x	space variable
z	inner space variable.

Greek symbols

α^{-1}	molar mass of the mixture
γ	ratio of the specific heats
δ	boundary layer thickness
ε	small parameter
η	small parameter
κ	thermal diffusivity
ν	kinematic viscosity
ρ	specific mass of the bulk gaseous phase
ρ_0	initial value of the specific mass
ρ^e_A	equilibrium value of the partial specific mass of A at the interface.

Superscripts and subscript

$(\ )'$	dimensional variable
$(\bar{\ })$	outer variable
$(\tilde{\ })$	inner variable
$(\ )_A$	property related to specie A.

$$\rho_t + (\rho u)_x = 0 \tag{1}$$

$$\rho u_t + \rho u u_x = -\gamma^{-1} P_x + \tfrac{4}{3}\varepsilon u_{xx} + S \tag{2}$$

$$\frac{\rho}{\gamma-1}(T_t + uT_x) = -Pu_x + \varepsilon\left\{\frac{\gamma}{\gamma-1} Pr^{-1}\, T_{xx} + \tfrac{4}{3}\gamma (u_x)^2\right\} \tag{3}$$

$$P = \rho T \tag{4}$$

where ρ is the specific mass, u the velocity and T the temperature. The initial reference state is ρ'_0, $u'_0 = 0$ and $T' = T'_0$. ε is the ratio $Pr\ (t'_a/t'_d)$ of the acoustic time t'_a to the diffusion time L^2/κ where κ is the thermal diffusivity. $Pr = \kappa/\nu$, where ν is the kinematic viscosity, and Pr the Prandtl number. S is a momentum source term normalized with respect to a characteristic value $S^* = \rho'_0 C'^2_0/L$.

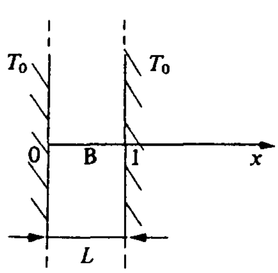

FIG. 1. One-dimensional slot filled with a pure gas B and non-catalytic walls.

The boundary conditions are

$$u = 0,\quad T = 1 \quad \text{at} \quad x = 0.1 \tag{5}$$

and the initial conditions correspond to the rest and thermodynamic equilibrium

$$u = 0,\quad T = P = \rho = 1 \quad \text{at} \quad t = 0. \tag{6}$$

The source term S is considered to be

$$S = \rho A \sin \omega t \quad t \geqslant 0 \tag{7}$$

where

$$A = \frac{A'L}{C'^2_0} \quad \text{and} \quad \omega' t'_a. \tag{8}$$

2.2. *Outer solution*

The main scaling laws and domain of the asymptotic analysis are summarized on Fig. 2.

2.2.1. *Scaling*. The considered values for the amplitude and frequency of the residual acceleration field ($g = 10^{-2} g_0$, $f' = 50$ Hz) lead to

$$A = 10^{-7} \quad \text{and} \quad \omega = 0.1. \tag{9}$$

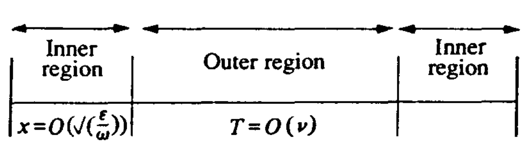

FIG. 2. One-dimensional slot filled with a perfect binary gas mixture A + B and with a growth interface at $x = 0$.

Equation (9) for ω indicates that the time scale of the perturbation $\bar{t} = \omega t$ is longer than the acoustic scale t but shorter than the diffusive scale $\tau = \varepsilon t$. Solution for equations (1)–(7) is looked for on the time scale of the perturbation by setting the new time scale to

$$\bar{t} = \omega t. \tag{10}$$

As equation (9) for A indicates that the source term is a small perturbation, the coupled thermodynamic variables are looked for under the form of the following asymptotic expansions:

$$P = 1 + v\bar{P}(x, \bar{t}) + o(v) \tag{11}$$

$$T = 1 + v\bar{T}(x, \bar{t}) + o(v) \tag{12}$$

$$\rho = 1 + v\bar{\rho}(x, \bar{t}) + o(v) \tag{13}$$

while the velocity is expanded as

$$u = \eta\bar{u}(x, \bar{t}) + o(\eta) \tag{14}$$

where v and η are to be determined.

The continuity equation (1), written in terms of the perturbed variables

$$v\omega\bar{\rho}_{\bar{t}} + \eta\bar{u}_x = 0 \tag{15}$$

suggests to consider for u the scaling

$$\eta = v\omega$$

and substitution of this scaling in equations (1)–(7) gives

$$\bar{P} = \bar{T} + \bar{\rho} \tag{16}$$

$$\bar{\rho}_{\bar{t}} = -\bar{u}_x \tag{17}$$

$$\bar{T}_{\bar{t}} = -(\gamma - 1)\bar{u}_x. \tag{18}$$

In order to enable the source term to drive a flow through the pressure gradient, the scaling for P, ρ and T must be

$$v = A \tag{19}$$

to give

$$\bar{P}_x = \gamma \sin \bar{t}. \tag{20}$$

The perturbed flow field is thus described by equations (16)–(18) and (20) together with boundary conditions

$$\bar{T} = \bar{u} = 0 \quad \text{at} \quad x = 0.1 \tag{21}$$

and initial conditions

$$\bar{u} = \bar{T} = \bar{P} = \bar{\rho} = 0 \quad \text{at} \quad \bar{t} = 0. \tag{22}$$

2.2.2. *Core solution.* The solution of equations (16)–(20) with conditions (21) and (22) is straightforward and can be written as

$$\bar{u}(x, \bar{t}) = \frac{x}{2}(1 - x)\cos \bar{t} \tag{23}$$

$$\bar{P}(x, \bar{t}) = \gamma(x - \tfrac{1}{2})\sin \bar{t} \tag{24}$$

$$\bar{\rho}(x, \bar{t}) = (x - \tfrac{1}{2})\sin \bar{t} \tag{25}$$

$$\bar{T}(x, \bar{t}) = (\gamma - 1)(x - \tfrac{1}{2})\sin \bar{t}. \tag{26}$$

Solution (27) for T does not satisfy condition (21) at $x = 0.1$. This indicates that solution (23)–(26) is an outer (core) solution for equations (1)–(7). This means physically that on the short time scale $\bar{t}$ of the perturbation (compared to the diffusive one), the pressure work source term $\bar{u}_x$ in the energy equation has no time to diffuse.

A thermal boundary layer exists in the neighbourhood of $x = 0.1$ even though the boundary conditions are satisfied for u. An inner solution has to be looked for in these regions.

2.3. *Boundary layer solution*

The boundary layers near $x = 0, 1$ are very similar and only the solution in the neighbourhood of $x = 0$ is treated here.

2.3.1. *Boundary layer scaling and governing equations.* In the core, the outer expansion holds

$$T = 1 + v\bar{T} + o(v). \tag{27}$$

So that

$$\lim_{x \to 0} T(x, \bar{T}) = 1 + v\bar{T}(0, \bar{t}) + o(v)$$

which suggests to look for the boundary layer solution under the form of the following inner asymptotic expansion:

$$T = 1 + v\tilde{T}(z, \bar{t}) + o(v) \tag{28}$$

where z is the boundary layer variable defined by

$$z = \frac{x}{\delta} \tag{29}$$

in which δ is the boundary layer thickness to be determined. Similar arguments suggest to look for the inner solution for P and ρ under the following expansions:

$$P = 1 + v\tilde{P}(z, \bar{t}) + o(v) \tag{30}$$

$$\rho = 1 + v\tilde{\rho}(z, \bar{t}) + o(v). \tag{31}$$

On the other hand, solution (23) being regular for $x \to 0$

$$\lim_{x \to 0} u(x, \bar{t}) = \tfrac{1}{2}v\omega\delta \cos \bar{t}\, z + O(v\omega\delta^2)$$

gives the scaling for u in the boundary layer and the inner asymptotic expansion

$$u = v\omega\delta\tilde{u}(z, \bar{t}) + o(v\omega\delta). \tag{32}$$

The matching conditions for $\tilde{T}(z, \bar{t})$ are

$$\lim_{z \to \infty} \tilde{T}(z, \bar{t}) = \lim_{x \to 0} \bar{T}(x, \bar{t}) = \tfrac{1}{2}(1 - \gamma)\sin \bar{t}.$$

Substituting inner expansions (28) and (30)–(32) in the Navier–Stokes equations, gives

$$\tilde{P} = \tilde{T} + \tilde{\rho} \tag{33}$$

$$\tilde{P}_z = 0 \tag{34}$$

$$\tilde{\rho}_{\bar{t}} = -\tilde{u}_z \tag{35}$$

and

$$v\omega\tilde{T}_{\bar{t}} = v\omega(\gamma-1)\tilde{u}_z + \gamma\, Pr^{-1}\,\varepsilon v\tilde{T}_{zz}/\delta^2$$

which gives the boundary layer thickness δ by matching the temporal term with the diffusion one, that is to say

$$\delta = \sqrt{\left(\frac{\varepsilon}{\omega}\right)} \tag{36}$$

which leads to the energy equation in the boundary layer

$$\tilde{T}_{\bar{t}} = -(\gamma-1)\tilde{u}_z + \gamma\, Pr^{-1}\,\tilde{T}_{zz}. \tag{37}$$

The governing equations for the boundary layer solutions are thus equations (33)–(35) and (37), with:

boundary conditions

$$\tilde{u} = \tilde{T} = 0 \quad \text{at} \quad t = 0; \tag{38}$$

matching conditions

$$\tilde{T}(z,\bar{t}) \to \frac{1-\gamma}{2}\sin\bar{t} \quad \text{at} \quad z\to\infty \tag{39}$$

$$\tilde{P}(z,\bar{t}) \to -\frac{\gamma}{2}\sin\bar{t} \quad \text{at} \quad t\to\infty; \tag{40}$$

and initial conditions

$$\tilde{u} = \tilde{T} = \tilde{P} = \tilde{\rho} = 0 \quad \text{at} \quad t = 0. \tag{41}$$

2.3.2. *Boundary layer solution for T.* From equations (35), (34) and (33)

$$\tilde{u}_z = \tilde{T}_{\bar{t}} - \tilde{P}_{\bar{t}}. \tag{42}$$

Substituting equation (42) into equation (37) we obtain

$$\tilde{T}_{\bar{t}} = Pr^{-1}\,\tilde{T}_{zz} - \frac{\gamma-1}{2}\cos\bar{t} \tag{43}$$

with, for $\tilde{T}$, boundary condition (38), matching condition (39) and initial condition (41). By setting

$$\theta = \tilde{T} + \frac{\gamma-1}{2}\sin\bar{t}$$

governing equations for θ are

$$\begin{aligned} &\theta_{\bar{t}} = Pr^{-1}\,\theta_{zz} \\ &\theta = \frac{\gamma-1}{2}\sin\bar{t} \quad \text{at} \quad z = 0 \\ &\theta \to 0' \quad \text{at} \quad x\to\infty \\ &\theta = 0 \quad \text{at} \quad \bar{t} = 0. \end{aligned} \tag{44}$$

The Laplace transform gives the solution for θ and thus for $\tilde{T}(z,\bar{t})$

$$\tilde{T}(z,\bar{t}) = \frac{\gamma-1}{2}\left\{ e^{-\sqrt{(Pr/2)}z}\sin\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right) + \frac{1}{\pi}\int_0^\infty \frac{e^{-u\bar{t}}}{1+u^2}\sin(z\sqrt{(Pr\,u)})\,\mathrm{d}u - \sin\bar{t}\right\}. \tag{45}$$

The first term in equation (45) represents a damped travelling wave, while the second term represents a transient term which goes to zero as $\bar{t}\to\infty$. The third term matches $\tilde{T}(z,\bar{t})$ with the core temperature field.

2.3.3. *Boundary layer solution for the velocity.* From equation (35)

$$\tilde{u}_z = \tilde{T}_{\bar{t}} - \tilde{P}_{\bar{t}}$$

and invoking equation (37) leads to

$$\tilde{u}_z = Pr^{-1}\,\tilde{T}_{zz} - \gamma^{-1}\tilde{P}_{\bar{t}}(\bar{t}) \tag{46}$$

which gives under integration

$$\tilde{u}(z,\bar{t}) = Pr^{-1}\,\tilde{T}_z - \frac{z}{\gamma}\tilde{P}_{\bar{t}}(\bar{t}) + C^{te}.$$

Now, taking into account $\tilde{T}(z,\bar{t})$ to get $\tilde{T}_z$ and boundary condition (38) for u to get C^{te}, the solution for u, the transient being ignored, can be written as

$$\tilde{u}(z,t) = -\frac{\gamma-1}{2}(2Pr)^{-1/2}\left\{ e^{-\sqrt{(Pr/2)}z} \times\left\{\sin\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right)+\cos\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right)\right\} -\sin\bar{t}-\cos\bar{t}\right\} + \frac{z}{2}\cos\bar{t}. \tag{47}$$

The term $\frac{1}{2}z\cos\bar{t}$ in equation (47) is the driving velocity field as $x\to 0$ near the boundary. The other terms correspond to the expansion or the contraction of the thermal boundary layer under the external oscillating thermal solicitation.

Considering now $\lim_{z\to\infty} u(z,\bar{t})$, it becomes

$$\begin{aligned}\lim_{z\to\infty} u(z,\bar{t}) &= v\omega\delta\,\frac{\gamma-1}{2\sqrt{(2Pr)}}(\sin\bar{t}+\cos\bar{t}) \\ &\quad + \tfrac{1}{2}v\omega x\cos\bar{t} \\ &= \lim_{x\to 0}\bar{u}(x,\bar{t}) + O(v\omega\delta)\end{aligned}$$

which proves that the matching conditions are fulfilled up to the retained order $O(v\omega)$.

The extra term $O(v\omega\delta)$ represents an oscillating piston effect, analogous to that of ref. [3]. This piston effect will drive the next approximation under the form of a mechanical disturbance in the core flow, of order $O(v\omega\delta)$. The asymptotic sequence for the core expansion is thus

$$u = v\omega\bar{u}(x,\bar{t}) + v\omega\delta\bar{\bar{u}}(x,\bar{t}) + o(v\omega\delta). \tag{48}$$

2.3.4. *Boundary layer solution for ρ.* From equation (33), the solution for $\tilde{\rho}(z,\bar{t})$ is obviously

$$\tilde{\rho}(z,\bar{t}) = -\frac{\gamma-1}{2}e^{-\sqrt{(Pr/2)}z}\sin\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right) - \tfrac{1}{2}\sin\bar{t}. \tag{49}$$

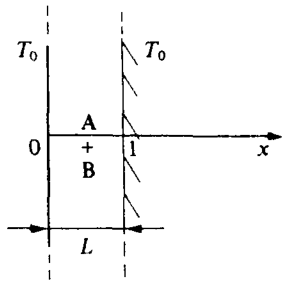

FIG. 3. One-dimensional slot filled with a binary mixture and bounded with a catalytic wall at $x = 0$.

3. BINARY GAS MIXTURE AND CATALYTIC WALL

3.1. *The model and governing equations*

The model is the same as in Section 1 except that the slot is filled with a binary perfect gas mixture, one component of which A, undergoes a phase change at $x = 0$ while the other, B, is inert (Fig. 3).

The Navier–Stokes equations are coupled with the species transport equations which can be written as

$$W_t + \left(u - \frac{\varepsilon}{Le\ Pr}\frac{\rho_x}{\rho}\right)W_x = \frac{\varepsilon}{Le\ Pr}W_{xx} \qquad (50)$$

$Le = \kappa/D$, where D is the diffusion coefficient, Le the Lewis number, and W the weight fraction of A.

The state equation is that of a perfect gas mixture

$$P = \rho\alpha T$$

where

$$\alpha = \frac{\alpha'}{\alpha'_0} \quad \text{and} \quad \alpha = \frac{M_B - M_A}{M_A M_B}W + \frac{1}{M_B}$$

where M_A and M_B are the molar masses of A and B.

The initial and boundary conditions are the same as in Section 1, except for the velocity at $x = 0$ for which the species balance can be written as

$$u = \frac{\varepsilon}{Le\ Pr}\frac{W_x}{\left(\dfrac{W_0^e}{\rho} - 1\right)} \quad \text{at} \quad x = 0 \qquad (51)$$

for which it has been supposed that the transfer at the interface is diffusion limited [8]. W_0^e is the equilibrium weight fraction defined by

$$W_0^e = \frac{\rho_A^e(T)}{\rho}$$

where $\rho_A^e(T)$ is the equilibrium value of the partial specific mass of A which is dependent on the interface temperature.

As the end at $x = 1$ is closed the boundary condition for W is of the Neuman type

$$W_x = 0 \quad \text{at} \quad x = 1$$

$$W = \frac{W_0^e}{\rho} \quad \text{at} \quad x = 0. \qquad (52)$$

The initial condition for W is the thermodynamic equilibrium

$$W = W_0^e \quad \text{at} \quad \bar{t} = 0. \qquad (53)$$

The governing equations for the binary mixture with catalytic walls are thus equations (1)–(4) and (50) with boundary conditions (5), (51) and (52), and initial conditions (6) and (53).

3.2. *Outer solution*

3.2.1. *Scaling and governing equations.* In the same way as in Section 2.3.1, the solution is looked for on the time scale of the perturbation. Taking into account expansions (11)–(13) and (14) and (15) as well as

$$W = W_0^e + \nu\bar{W}(x, \bar{t}) + o(\nu) \qquad (54)$$

the governing equations for the outer solution are

$$\bar{P}_x = \gamma \sin \bar{t}$$

$$\bar{P} = \bar{T} + \bar{\rho} + \xi\bar{W}$$

$$\xi = \frac{k_1}{k_1 W_0^e k_2}, \quad k_1 = \frac{M_B - M_A}{M_A M_B}, \quad k_2 = \frac{1}{M_B}$$

$$\bar{\rho}_{\bar{t}} = -\bar{u}_x$$

$$\bar{T}_{\bar{t}} = -(\gamma - 1)\bar{u}_x$$

$$\bar{W}_{\bar{t}} = 0 \qquad (55)$$

with boundary conditions

$$\bar{T} = 0, \quad \bar{W} = -W_0^e\bar{\rho}(0, \bar{t}), \quad \bar{u}(0, \bar{t}) \quad \text{at} \quad x = 0$$

$$\bar{T} = 0, \quad \bar{W}_x = 0, \quad \bar{u} = 0 \quad \text{at} \quad x = 1 \qquad (56)$$

and initial conditions

$$\bar{T} = 0, \quad \bar{u} = 0, \quad \bar{W} = W_0^e \quad \text{at} \quad \bar{t} = 0.$$

The solution for equation (54) for $\bar{W}$ is

$$\bar{W}(x, \bar{t}) = 0 \qquad (57)$$

and boundary condition (56) for u becomes

$$\bar{u} = 0 \quad \text{at} \quad x = 0. \qquad (58)$$

Equation (57) means that on the time scale of the perturbation which is short compared to the diffusion one, no diffusion has time to occur in the bulk.

However, as boundary condition (56) for W is not satisfied by the core solution (57), this core solution is not regular for $x = 0$. The asymptotic expansion (54) for W is not uniform in the neighbourhood of $x = 0$.

According to the matched asymptotic expansion technique, an inner expansion must be introduced, which is constructed with an inner variable z defined by

$$z = \frac{x}{\delta(\varepsilon)}, \quad \delta(\varepsilon) \to 0 \quad \text{when} \quad \varepsilon \to 0 \qquad (59)$$

where $\delta(\varepsilon)$ is given by the least degeneracy principle applied to the whole diffusion equation written in terms of the inner variable z.

The remainder of the core solution for u, P, T and ρ, are here too given by equations (23), (24), (27) and (26).

3.3. *Boundary layer solution at* $x = 0$

From solution (27) for $T(x, t)$ and boundary conditions (21) and (56) for $W(x, t)$ two boundary layers exist for T at $x = 0$, 1 and for W at $x = 0$. As we are essentially interested in what follows in the behaviour of the interface, the boundary layer solution near $x = 0$ is only studied.

3.3.1. *Scaling and governing equations.* In the same way as in Section 2.3.1, the outer expansion scaling for the thermodynamic variables leads to inner expansions (28), (30) and (31) for T, P and ρ. Now considering boundary condition (56) for W, it appears that the inner expansion for W is

$$W = W_0^e + \nu \tilde{W}(z, \bar{t}) + o(\nu). \tag{60}$$

The least degeneracy principle which matches the temporal and diffusive terms in equation (50) written in terms of the inner variables, that is to say for the functions

$$W(x, \bar{t}) = W(\delta_t, \bar{t}) = \tilde{W}(z, \bar{t})$$

suggests to consider

$$\delta = \left(\frac{\varepsilon}{\omega}\right)^{1/2} \tag{61}$$

and leads to the inner boundary condition scaling for u

$$u = \nu\sqrt{(\varepsilon\omega)}\frac{\tilde{W}_z}{Pr\, Le\,(W_0^e - 1)} \quad \text{at} \quad x = 0 \tag{62}$$

which suggests to consider as the inner expansion for u

$$u = \nu\sqrt{(\varepsilon\omega)}\tilde{u}(z, \bar{t}) + o(\nu\sqrt{(\varepsilon\omega)}). \tag{63}$$

This scaling matches with the outer solution as

$$\lim_{x \to 0} u(x, \bar{t}) = \tfrac{1}{2}\nu\omega(\delta z \cos \bar{t} + o(\delta)). \tag{64}$$

This means physically that the thermal disturbance in the core caused by the pressure work induces velocity disturbances in the boundary layer which are of the same order of magnitude as the interface velocity induced by density disturbances; as a matter of fact, substituting equation (61) for δ in equation (64) leads to

$$\nu\omega\delta = \nu\sqrt{(\varepsilon\omega)}.$$

Under these scalings, the governing equations for the boundary layer solution near $x = 0$ are

$$\begin{aligned}
&\tilde{P} = \tilde{\rho} + \tilde{T} + \xi\tilde{W}, \quad \tilde{P} \equiv \tilde{P}(\bar{t}) \\
&\tilde{\rho}_{\bar{t}} = -\tilde{u}_z \\
&\tilde{T}_{\bar{t}} = \gamma\, Pr^{-1}\, \tilde{T}_{zz} - (\gamma - 1)\tilde{u}_z \\
&\tilde{W}_{\bar{t}} = \frac{1}{Le\, Pr}\tilde{W}_{zz}
\end{aligned} \tag{65}$$

with boundary conditions

$$\tilde{u} = \frac{\tilde{W}_z}{Pr\, Le\,(W_0^e - 1)}, \quad \tilde{W} = -W_0^e \tilde{\rho}(0, \bar{t}), \quad \tilde{T} = 0 \quad \text{at} \quad z = 0 \tag{66}$$

matching conditions at $z \to \infty$

$$\begin{aligned}
&\tilde{W}(z, \bar{t}) \to 0 \\
&\tilde{T}(z, \bar{t}) \to \frac{\gamma - 1}{2}\sin \bar{t} \\
&\tilde{P}(z, \bar{t}) \to -\frac{\gamma}{2}\sin \bar{t} \\
&\tilde{u}(z, \bar{t}) \to \frac{z}{2}\cos \bar{t}
\end{aligned} \tag{67}$$

and initial conditions

$$\tilde{W} = \tilde{T} = \tilde{u} = \tilde{\rho} = \tilde{P} = 0 \quad \text{at} \quad \bar{t} = 0. \tag{68}$$

3.3.2. *Boundary layer solutions for W.* The boundary layer solution is obtained by the Laplace transform technique from equations (65)–(68) for W

$$\tilde{W}(z, \bar{t}) = \frac{W_0^e}{2}\left\{e^{-\sqrt{((Pr\,Le)/2)}z}\sin\left(\bar{t} - \sqrt{\left(\frac{Pr\,Le}{2}\right)}z\right) + \frac{1}{\pi}\int_0^\infty \frac{e^{-u\bar{t}}}{1 + u^2}\sin(z\sqrt{(Le\,Pr\,u)})\,du\right\}. \tag{69}$$

The second term represents a transient state and the first term represents a damped travelling concentration wave. The penetration length for the permanent state

$$l_p = \left(\frac{2}{Le\,Pr}\right)^{1/2}\delta$$

is of the order of the boundary layer thickness δ, and the velocity of the travelling wave

$$v_t = \left(\frac{2}{Le\,Pr}\right)^{1/2}\delta\omega$$

is larger than the diffusion velocity.

3.3.3. *Boundary layer solution for T.* Substituting for $\tilde{u}_z$ in equation (65) for $\tilde{T}$, equation (65) for $\tilde{\rho}$ and $\tilde{P}$, it gives

$$\tilde{T}_{\bar{t}} = Pr^{-1}\,\tilde{T}_{zz} + \frac{\gamma - 1}{2}(\tilde{P}_{\bar{t}} - \xi\tilde{W}_{\bar{t}}). \tag{70}$$

In order to reach a simplified description of the problem, we choose to ignore the solutal effects, that is to say that we suppose $M_A = M_B$ in order to have

$$\xi = 0. \tag{71}$$

In these conditions, equation (65) and matching conditions (67) for $\tilde{P}$, equation (65) for $\tilde{T}$ and conditions (66) and (67) define the same problem as in Section 2.3.2 and solution for $\tilde{T}$ in the case where $M_A = M_B$ ($\xi = 0$) is thus equation (45).

3.3.4. *Boundary layer solution for u.* From equation (65) for $\tilde{u}_z$ and $\tilde{P}$, and taking into account condition (71), it comes under integration of the obtained equation over z

$$\tilde{u}(z,\bar{t}) = Pr^{-1}\,\tilde{T}_z - \frac{z}{\gamma}\,\tilde{P}_{\bar{t}} + C^{\text{ste}}. \tag{72}$$

From solution (45) for $\tilde{T}(z,t)$, boundary condition (66) for $\tilde{u}$ at $z = 0$ and solution (69) for $\tilde{W}$, solution for $\tilde{u}$ is, if the transient term is ignored

$$\tilde{u}(z,\bar{t}) = -\frac{\gamma-1}{2}(2Pr)^{-1/2}\Bigg\{e^{-\sqrt{(Pr/2)}z} \times\left\{\sin\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right)+\cos\left(\bar{t}-\sqrt{\left(\frac{Pr}{2}\right)}z\right)\right\} -(\sin\bar{t}+\cos\bar{t})\Bigg\} - \frac{W_0^e}{2Le\sqrt{(2Pr)}(W_0^e-1)}(\sin\bar{t}+\cos\bar{t})+\frac{z}{2}\cos\bar{t}. \tag{73}$$

The first two terms in equation (73) represent the contribution of the thermal disturbances in the boundary layer, to the fluid velocity.

The third term is the contribution of the phase change at the interface at $z = 0$; the fourth term is the driving velocity field in the bulk as $x \to 0$.

Taking now the limit when $z \to \infty$ for $u(z,t)$ with $x \ll 1$ and $x \to 0$, it becomes

$$\lim_{\substack{z\to\infty\\ \varepsilon\to 0\\ x\ll 1}} u(z,\bar{t}) = v\omega\frac{x}{2}\cos\bar{t}+v\omega\delta(2Pr)^{-1/2}(\sin\bar{t}+\cos\bar{t}) \times\left\{\frac{\gamma-1}{2}-\frac{W_0^e}{2Le(W_0^e-1)}\right\} = \lim_{x\to\infty} u(x,\bar{t})+O(v\omega\delta).$$

The matching conditions are fulfilled and the term $O(v\omega\delta)$ represents the velocity at the edge of the boundary layer which gives the order of magnitude of the next term in the outer expansion for u. This oscillating piston effect will drive a new disturbance of mechanical origin in the core; the driving velocity U_P for this perturbation is

$$U_P = U(0,\bar{t}) = (2Pr)^{-1/2}(\sin\bar{t}+\cos\bar{t})\Bigg(\underbrace{\frac{\gamma-1}{2}}_{\text{I}} - \underbrace{\frac{W_0^e}{2Le(W_0^e-1)}}_{\text{II}}\Bigg).$$

Table 1. Comparison between velocity perturbations and Stefan velocity for two growth conditions, L = 10 cm

	Case 1	Case 2
C'_0 (m s^{-1})	130	130
P'(Torr)	0.1	1
D' (cm^2 s^{-1})	120	15.4
U'_{Stefan} (μm s^{-1})	4.86×10^3	0.440
U' (μm s^{-1}): interface	0.835	0.282
bulk	7.5	7.5

Term I represents the overall expansion of contraction of the boundary layer under the thermal disturbances of the core flow caused by the work of pressure forces. Term II represents the expansion or contraction of the boundary layer due to the evaporation or condensation of the crystal under mechanical (density) disturbances in the bulk.

It must be emphasized that no solutal effect is present because $\xi = 0$: when $\xi \neq 0$ [9] diffusion of a heavy specie for example will cause an increase in ρ and a contraction of the boundary layer and this phenomenon is not taken into account here.

4. ORDER OF MAGNITUDE

It has been shown that the order of magnitude of the velocity in the bulk at the interface was the same as that of the velocity in the boundary layer, that is to say

$$u = O(v\sqrt{(\varepsilon\omega)})$$

while the velocity perturbation in the bulk is of the order of $v\omega'$. Comparison of these velocity perturbations with the typical velocity encountered in HgI_2 growth experiments are summarized on Table 1. For systems for which the pressure is 0.1 Torr or less, growth rates between 1 and 0.5 mm day^{-1} have been observed [10] and are represented by case 1.

For pressure increases of about 1 Torr of argon, the growth rates are very low because the diffusion strongly decreases and growth velocities as low as 10^{-5} mm day^{-1} have been observed and are represented by case 2.

It has been supposed that the order of magnitude of the velocity perturbation is independent of the initial conditions. As a matter of fact, in order to compare with typical growth rates, a different initial value problem should have been studied for which the basic flow is a Stefan flow between a source and a sink. Nevertheless it can be seen in Table 1 that for low pressure systems, both interfacial and bulk perturbations are negligible compared to the Stefan wind. However, for case 2, the bulk perturbation is one decade higher than the basic flow while the interfacial velocity perturbation is of the same order as the Stefan velocity. This points out that for high pressure experiments for which the growth velocity is very small, perturbations caused by g-jitters may be taken into account. On the other hand, in some experiments such as epitaxial growth of germanium, the etching of the substrate to which corresponds very small velocities [11, 12] may come from such mechanical perturbations.

5. CONCLUSION

The core and boundary layer solutions have been obtained for the response of a solid–gas growth interface to small periodic disturbances of a basically zero gravity field.

It has been shown that for typical values of the perturbations in the gravity encountered onboard spacecrafts in low earth orbit, say $10^{-2}g_0$ and $f = 50$ Hz, the perturbation in the core velocity is of the order of 10^{-4} cm s^{-1} while the perturbation in the velocity at the interface may be of the same order as the Stefan velocity for high pressure growth conditions when considering mercury iodide typical values. This means that unless very small growth velocities are considered as it is the case for some epitaxial growth processes, g-jitters, as those encountered in spacecrafts have negligible effects on the mass transfer. However, the perturbations in weight fraction at the interface are of the order of 10^{-7}; under steady state growth, and not, as in the present case, under thermodynamic equilibrium reference state, will these oscillations produce significant oscillations in dopant concentration in the grown crystal. This would be interesting to check experimentally.

On the other hand, it would be interesting to look at the influence of the molar mass difference on the contraction or expansion of the boundary layer. In the same way, non-equilibrium transfer could be introduced through mixed type boundary conditions for the species equations and would lead to a physically more significant description. Moreover, a generalization of this study to higher frequency for which acoustic modes could be excited would certainly lead to interesting new features concerning interaction between acoustics and crystal growth.

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REPONSE D'UNE INTERFACE DE SOLIDIFICATION SOLIDE–GAZ A UN CHAMP D'ACCELERATION HOMOGENE DEPENDANT DU TEMPS

Résumé—La technique des développements asymptotiques raccordées est utilisée pour analyser les solutions intérieures et extérieures des équations de Navier–Stokes 1-D instationnaire et compressible pour étudier le transport à une interface dû à un champ d'accélération faible et variant sinusoïdalement dans le temps de manière lente. Il est montré que: (a) le transfer de masse à l'interface est gouverné par la dépendance temporelle de la masse spécifique du gaz sur l'interface; (b) la diffusion des espèces intervient dans une couche limite mince située devant l'interface de croissance; (c) à moins de considérer des vitesses de croissance très faibles comme c'est le case pour certains procédés de croissance épitaxiale, les g-jitters tels qu'on les rencontre à bord des véhicules spatiaux ont des effets négligeables sur le transfert de masse à un interface de croissance solide gaz.

VERHALTEN EINER WACHSENDEN PHASENGRENZFLÄCHE ZWISCHEN FESTEM UND GASFÖRMIGEM ZUSTAND IN EINEM HOMOGENEN, ZEITLICH VERÄNDERLICHEN BESCHLEUNIGUNGSFELD

Zusammenfassung—Das Verfahren der angepaßten asymptotischen Entwicklungen wird zur Lösung der kompressiblen, instationären, reibungsbehafteten Navier–Stokes-Gleichungen für den Kern- und Grenzschichtbereich verwendet. Damit sollen die Transportvorgänge an der Grenzfläche beim Kristallwachstum untersucht werden, die sich infolge eines homogenen, zeitlich sinusförmig und langsam veränderlichen Beschleunigungsfeldes ergeben. Es zeigt sich folgendes: (a) Der Stofftransport an der Grenzfläche wird von der Zeitabhängigkeit der spezifischen Masse an der isothermen aktiven Grenzfläche bestimmt; (b) in einer dünnen Grenzschicht unmittelbar vor der Grenzfläche tritt Diffusion aufgrund einer gedämpften, wandernden Konzentrationswelle auf; (c) außer bei sehr kleinen Wachstumsgeschwindigkeiten, wie sie bei einigen epitaxialen Wachstumsvorgängen vorkommen, haben g-Gitter einen vernachlässigbaren Einfluß auf den Stofftransport an einer fest–gasförmigen Wachstumsoberfläche.

ВЛИЯНИЕ ОДНОРОДНОГО НЕСТАЦИОНАРНОГО ПОЛЯ УСКОРЕНИЯ НА РОСТ ГРАНИЦЫ РАЗДЕЛА ТВЕРДОЕ ТЕЛО–ГАЗ

Аннотация—Методом сращивания асимптотических разложений анализируются решения уравнений Навье–Стокса для нестационарного течения сжимаемой вязкой жидкости в ядре и пограничном слое при исследовании процесса переноса у границы раздела, растущей в результате действия однородного, синусоидального во времени и медленно изменяющегося слабого поля силы тяжести. Показано, что: (а) массоперенос описывается временной зависимостью удельной массы у изотермической активной границы раздела; (б) диффузия происходит в тонком пограничном слое перед границей раздела в виде затухающей бегущей волны концентрации; (в) за исключением случаев очень низких скоростей роста, имеющих место при некоторых процессах эпитаксиального роста, наблюдаемые на космических кораблях изменения ускорения оказывают незначительное влияние на массоперенос у растущей границы раздела твердое тело–газ.